Experiment: Operational Amplifier

1 General Description

An operational amplifier (op-amp) is defined to be a high gain differential amplifier. When using the op-amp with other mainly passive elements, op-amp circuits with various characteristics result. Like the transistor, the op-amp belongs to the standard elements of circuit design [1].

In electronic circuit diagrams the op-amp is drawn as block diagram symbol in accordance to Fig. 1. Terminals for operating voltage, offset voltage compensation, frequency response compensation etc. frequently are not drawn.

![Block diagram symbol of the operational amplifier](image)

When designing circuits with op-amps, they normally are based on the "ideal operational amplifier", whose characteristics are represented in Table 1. For comparison the data areas of real op-amps were listed as well.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>ideal op-amp</th>
<th>real op-amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC voltage difference amplification $V_o$</td>
<td>$\infty$</td>
<td>$5,000 \leq V_o \leq 5,000,000$</td>
</tr>
<tr>
<td>common mode rejection CMRR</td>
<td>$\infty$</td>
<td>$90 , \text{dB} \leq \text{CMRR} \leq 140 , \text{dB}$</td>
</tr>
<tr>
<td>Input impedances $Z_p$ and $Z_N$</td>
<td>$\infty$</td>
<td>$10^6 , \Omega \leq Z_p, Z_N \leq 10^{15} , \Omega$</td>
</tr>
<tr>
<td>Input currents $I_p$ and $I_N$</td>
<td>0</td>
<td>$0,1,\text{pA} \leq I_p, I_N \leq 0,5,\mu\text{A}$</td>
</tr>
<tr>
<td>Output resistance $Z_o$</td>
<td>0</td>
<td>$5 , \Omega \leq Z_o \leq 500,\Omega$</td>
</tr>
<tr>
<td>Slew rate $SR$</td>
<td>$\infty$</td>
<td>$0,2 , \text{V/}\mu\text{s} \leq SR \leq 50 , \text{V/}\mu\text{s}$</td>
</tr>
<tr>
<td>Transit frequency $f_T$</td>
<td>$\infty$</td>
<td>$0,1 , \text{MHz} \leq f_T \leq 60 , \text{MHz}$</td>
</tr>
</tbody>
</table>

Table 1: Characteristics of ideal and real operational amplifiers
1.1 Basic Circuits

Circuits with op-amps mainly are based on two basic circuits, the inverting amplifier according to Fig. 2a and the non-inverting amplifier according to Fig. 2b.

![Basic circuits with operational amplifiers](image)

**Fig. 2: Basic circuits with operational amplifiers**

a) inverting amplifiers  
   b) non-inverting amplifier

1.1.1 Calculation of the Inverting Amplifier

The following equations result when setting mesh and node equations according to Fig.2a:

\[
U_{a_0} = V_0 \cdot U_d \\
Z_u = 0 \rightarrow U_u = U_{a_0}
\]

\[
I_1 = \frac{U_1 + U_{a_0}}{R_1} = \frac{U_1 + U_{a_0}}{V_0} \cdot \frac{V_0}{R_1}
\]

\[
I_2 = \frac{U_a + U_{a_0}}{R_N} = \frac{U_a + U_{a_0}}{V_0} \cdot \frac{V_0}{R_N}
\]

\[
I_N = \frac{U_{a_0}}{V_0 \cdot Z_v} \quad I_1 + I_2 + I_N = 0
\]
The gain $V_i$ of the inverting op-amp circuit is:

$$V_i = \frac{U_a}{U_1} = -V_0 \frac{1}{1 + (V_0 + 1) \frac{R_1}{R_N} + \frac{R_i}{Z_c}}$$  \hspace{1cm} (1.1)

In Eq. (1.1) $\frac{R_i}{Z_c}$ can be neglected. When solving for the gain we receive:

$$|V_i| = \frac{1 - k}{\frac{1}{V_0} + k},$$  \hspace{1cm} (1.2)

using $k = \frac{R_i}{R_i + R_N} = \text{feedback factor}$. For a large differential gain $V_0$, $V_i$ becomes:

$$V_i \bigg|_{V_0 \to \infty} = \frac{1}{k} \cdot (1 - k) = 1 - \frac{1}{k} = -\frac{R_N}{R_i}. \hspace{1cm} (1.3)$$

### 1.1.2 Calculation of the Non-Inverting Amplifier

For the non-inverting amplifier according to Fig. 2b we receive:

$$V_N = V_0 \frac{1}{1 + kV_0}. \hspace{1cm} (1.4)$$

If the differential gain $V_0$ is very large, Eq. (1.4) simplifies to:

$$V_i \bigg|_{V_0 \to \infty} = \frac{1}{k} \quad \text{with} \quad k = \frac{R_i}{R_i + R_N}. \hspace{1cm} (1.5)$$

### 1.2 Offset

The differential amplifier stages of the op-amps are implemented with bipolar transistors or field-effect transistors. For the operating point adjustment of these transistors base or gate currents are necessary. With MOSFETS, parasitic currents occur. Examples for $I_P$ and $I_N$ are given in Table 1. The linear average value of these currents is called idle or bias current:

$$I_B = 0.5 \cdot (I_P + I_N), \hspace{1cm} (1.6)$$

while the difference is called offset current:

$$I_{off} = I_P - I_N. \hspace{1cm} (1.7)$$
The current $I_N$ from Fig. 2 through the resistors $R_1$ and $R_N$ causes voltages acting as additional input voltages. For the inverting amplifier this influence can be compensated by putting a resistor $R_p$ onto the non-inverting input as seen in Fig. 7. With the current $I_P$ a further input voltage is generated. It compensates for the input voltage given as $I_N$, if $I_P$ and $I_N$ are equal and $R_p = R_1 || R_N$. The offset current $I_{OFF}$ prevents a complete alignment.

Although the differential voltage $U_d$ of a real operational amplifier is zero, DC voltage can occur at the output of the op-amp as well. In order to compensate this output voltage, DC voltage must be applied at the input. This offset voltage is called $U_{Off}$ which is in the order of some mV and subject to thermal drift. Besides the offset compensation circuits, which produce additional voltages in front of the operational amplifier, there are some op-amps with special pins to perform this compensation. They can be connected to a trimming potentiometer according to Fig. 3a and Fig. 3b.

Not all applications of operational amplifiers do rely on correct offset compensation (e.g., pure AC amplifiers with AC coupling). In these cases an offset is omitted.

![Fig. 3: Some possibilities of offset compensation](image)

1.3 Common-Mode Gain and Common-Mode Rejection

If both inputs of the op-amp are connected to the same voltage $U_{eg1}$, the differential voltage between both inputs is zero. In case of offset-compensation the output voltage $U_{ag1}$ should be zero as well. In reality this will not occur. This results in the definition of a so-called common-mode gain $V_{g1}$:

$$V_{g1} = \frac{U_{ag1}}{U_{eg1}}.$$  \hspace{1cm} (1.8)

The common mode rejection ratio $G$ relates the open-loop gain to the common-mode gain:

$$G = \frac{V_0}{V_{g1}}.$$  \hspace{1cm} (1.9)
1.4 Slew Rate

In a real op-amp both, output and internal voltages and currents are limited. Therefore, internal parasitic capacitors can only be loaded with finite currents. That is one reason for the output voltage change delay of an op-amp. The amplifier responds to an input voltage step with a ramp shaped output signal. This signal's gradient is called 'slew rate' \( SR \). The slew rate influences in particular the reaction to large signals: A sine output voltage of the amplitude \( \hat{U}_A \) and the frequency \( f \) can reach a maximum gradient:

\[
\frac{d}{dt} U_a(t) \bigg|_{\text{max}} = 2\pi f \hat{U}_A .
\]

For a given signal amplitude \( \hat{U}_A \) the maximum frequency \( f_{\text{max}} \) is limited by the slew rate \( SR \) to:

\[
f_{\text{max}} \leq \frac{SR}{2\pi \hat{U}_A} .
\]

When these boundaries are not respected, nonlinear distortions occur.

1.5 Frequency Response and Stability

Apart from the slew rate the small signal behavior of an op-amp limits the bandwidth where the circuits can be used. We have to consider that an op-amp usually consists of several amplifier stages (differential amplifier stage, intermediate stage, output stage). Each of these stages shows a low-pass behavior of first order with threshold frequency \( \omega_{g_n} \). The gain \( V \) of the op-amp can therefore be given in complex form:

\[
V(j\omega) = V_0 \frac{1}{1 + j \frac{\omega}{\omega_{g_1}}} \cdot \frac{1}{1 + j \frac{\omega}{\omega_{g_2}}} \cdot \frac{1}{1 + j \frac{\omega}{\omega_{g_3}}}
\]

The transit frequency is defined as the frequency \( f_T \) of an op-amp at which the gain drops below 1, i.e.:

\[
|V(2\pi f_T)| = 1 .
\]

This value can only be regarded as a very preliminary criteria for the selection of op-amps.

In Fig. 4, the frequency and phase response are drawn logarithmically with respect to the frequency (Bode diagram) and are approximated by line segments. This approximation makes it easier to understand how frequency and phase characteristics are constructed. The linear approximation is possible only with logarithmically scaled frequency and gain axis in dB.

1.5.1 Stability

For dimensioning op-amp - circuits a further characteristic of the op-amp - model is important.
The angle \( \arg\{V(j\omega)\} \) takes values between \( 0^\circ \) and \(-270^\circ \) for positive frequencies. A real feedback ratio \( k \) according to Eq. (1.2) to Eq. (1.5) leads to instability of the circuit, if no suitable measures are taken. For the loop gain of these op-amp circuits we can state:

\[
V'(j\omega) = kV_0 \frac{1}{(1 + \frac{\omega}{\omega_{g1}})} \frac{1}{(1 + \frac{\omega}{\omega_{g2}})} \frac{1}{(1 + \frac{\omega}{\omega_{g3}})}.
\]  

(1.14)

Instability occurs whenever the amount of the gain \( |V'(j\omega)| \) is greater or equal to 1, and the phase shift is large enough that the inverse feedback degenerates to a positive feedback, i.e. the amount of the phase shift \( |\arg\{V'(j\omega)\}| \) becomes smaller or equal \( 180^\circ \). An op-amp circuit is stable for all frequencies \( \omega \) that fulfill:

\[
|V'(j\omega)| < 1 \quad \text{for all } \omega \text{ with } |\arg\{V'(j\omega)\}| \geq 180^\circ \quad (1.15)
\]

or equivalently:

\[
|\arg\{V'(j\omega)\}| < 180^\circ \quad \text{for all } \omega \text{ with } |V'(j\omega)| \geq 1 \quad (1.16)
\]

The difference between \(-180^\circ\) and the actual angle for which \( |V'(j\omega)|=1 \) is called phase reserve \( \Phi_r \). It is a quantity which determines the transient behavior of an op-amp circuit.

![Fig. 4: Typical frequency and phase response of an operational amplifier depicted in a logarithmically scaled frequency and gain axis in dB(Bode diagram)](image)
## 1.5.2 Frequency Response Compensation

If we want to avoid instability of the op-amp circuit, frequency response compensation has to be applied. Essentially, two principles can be introduced.

The easiest method is “lag compensation”. Here the op-amp is followed by a low-pass filter with low cutoff frequency. This causes the gain to drop below unity before achieving the critical phase shift of -180°.

The open-loop gain results from:

\[
V'(j\omega) = V'(j\omega) \cdot \frac{1}{1 + j\frac{\omega}{\omega_k}}. \tag{1.17}
\]

The cutoff frequency \(f_k\) of the compensating low-pass depends on the feedback ratio \(k\). If \(k = 0\) (no feedback), then there is no need of compensation. On the other hand, if \(k = 1\) (full feedback), then \(f_k\) is to be selected at least in such a way that \(|V(j\omega)| = 1\), if \(\Phi = -180°\) is achieved. If this is fulfilled exactly, the op-amp would have no phase reserve and the amplifier circuit would operate on the boundary between stability and instability.

In Fig. 5, the low-pass compensation for a phase reserve \(\Phi\) in the case of \(0 < k < 1\) is drawn (solid line). A compensation with a phase reserve of 65° is considered to be the optimum in most cases.

Following Eq. (1.5), the non-inverting op-amp circuit reaches the amplification

\[
V_N = \frac{1}{k} > 1.
\]

The product \(|V(j\omega)k|\) equals to unity if \(|V(j\omega)| = 1\). According to Fig. 5, the compensation frequency \(f_k\) has to be chosen such that at the frequency \(f_T\) an angular phase shift of:

\[
\Phi = \Phi_r - 180° \quad \text{with} \quad \Phi_r = \text{phase reserve} \tag{1.18}
\]

is reached.

For the graphic construction of \(f_k\) we often can assume \(f_T > 10 \cdot f_k\). This means that at frequency \(f_T\) the compensating low-pass filter shifts the phase by - 90° additionally. The amplitude gain decreases by 20 dB/decade.
A few op-amps have a special pin, at which an external compensating capacitor can be attached, see Fig. 6. The resistor $R_x$ is integrated into the op-amp chip.

To determine the value of the compensating capacitor from Fig. 6, Eq.(1.9) is used:

$$C_x = \frac{1}{2\pi f_k R_x}.$$  \hspace{1cm} (1.19)
For most of all of op-amp chips, the compensation is not achieved by insertion of an additional low-pass of low cutoff frequency, but rather by shifting the cutoff frequencies $f_{g1}$ and $f_{g2}$ (Miller compensation). Here, the compensating capacitor is not attached to ground as shown in Fig. 6, but connected to two pins of the integrated circuit of the op-amp. This method of compensation makes use of the so-called "Miller-effect" of the capacitive feedback [2].

The first cutoff frequency $f_{g1}$ of the non-compensated op-amp is shifted to the frequency $f_{M1}$

$$f_{M1} = \frac{1}{2\pi R_{x1} C_k}$$  (1.20)

which causes no change in Eq.(1.19) to calculate $C_k$. However, $f_{g1}$ disappears in the Bode diagram of the compensated op-amp. The frequency $f_{g2}$ is shifted with $C_k$ to a higher frequency $f_{M2}$

$$f_{M2} = \frac{1}{2\pi R_{x2}(C_1 + \frac{C_1 C_2}{C_k})}.$$  (1.21)

Here, $R_{x1}$, $R_{x2}$, $C_1$ and $C_2$ are integrated in the op-amp IC, and could not be altered from outside. Their values are not found in data sheets. If these elements must be considered in the compensation, their values have to be determined by frequency response measurements. In Fig. 5, the frequency response curve for an op-amp using this compensation technique is drawn (dotted line). When comparing low-pass with frequency shift compensation, a larger phase reserve can be attained with this frequency shift, and thus a larger bandwidth of the amplifier can be used. In the experiment, an op-amp with "Miller-compensation" is used.

2 References


3 The Experiment

3.1 Equipment

1 dual channel oscilloscope 0 Hz ...10MHz
1 multimeter
1 function generator 10 Hz ... 5MHz
1 frequency counter
1 test circuit

3.2 Tasks

1) Before the experiment (this means: **at home**) the resistors $R_1$ and $R_N$ have to be calculated for the ideal and real op-amp for a DC voltage amplification of 40dB, both in the non-inverting and in the inverting circuit. Additionally, the compensating capacitor for a phase reserve $\Phi_r = 45^\circ$ has to be calculated, if the cutoff frequencies of the non-compensated op-amp are given by $f_{g1} = 16$kHz and $f_{g2} = 370$kHz. The open-loop gain is determined to 100dB and the resistance $R_x = 10^9 \Omega$. Calculate a low-pass compensation.

2) The offset voltage of the op-amp has to be measured with the circuit in Fig.7. The resistor $R_p$ is used only for compensation of the bias current influence. The equation for the offset voltage given here has to be derived from the theoretical equations.

![Fig. 7: Circuit for offset voltage detection](image)

\[
R_N = 10 \, k\Omega \\
R_1 = R_p = 100 \, \Omega \\
C_k = 33 \, pF \\
R_0 = 5,1 \, M\Omega \\
U_{Off} = U_a \frac{R_1}{R_N}
\]

3) In the circuit configuration in accordance to Fig. 7, a variable resistor has to be inserted into the socket marked "offset", and the offset voltage compensation has to be carried out. For zero indication a multimeter can be used.

4) With the circuit according to Fig. 8 the offset correction has to be done. Closed switches S1 and S2 and take into account the effect. For further experiments, the offset compensation should not be changed. Now the input currents as well as the offset current have to be determined. The following currents result:
5) According to Fig. 9, the common mode rejection ratio $G$ has to be measured for a frequency of 100 Hz. This is done by using the equation:

$$G_{dB} = 20 \log\left(\frac{U_1}{U_a} \cdot \frac{R_2}{R_1}\right)$$

This has to be done for $U_1 = 1V$, 5V and 8V.

6) With the circuit given in Fig. 10, the frequency response curve of an op-amp without feedback branch can be measured and drawn on logarithmically spaced paper in a Bode-diagram. Here, the open-loop gain $|V(j\omega)|$ in dB and the phase curve $\Phi(j\omega)$ should be approximated linearly. With an adjustable DC voltage of +/- 16 mV the output voltage drift always has to be corrected. It should be adjusted during the experiment continuously, so that the average value of the output voltage is zero. The output signal of the op-amp is connected with channel 2 of...
the oscilloscope. The frequency of the input signal has to be read from the frequency counter.

Always be aware of the connection between grounds in experimental setup, the wave generator and the oscilloscope.

The input voltage has to be determined in such a way that the op-amp output is not clipped. This can be detected on the oscilloscope, when the upper amplitude values of the output voltage are cut off (dynamic range exceeded) or when at higher frequencies the sine signal is distorted to a triangle signal (slew rate exceeded).

7) The experiment according to Fig. 10 has to be implemented with a capacitor of 22pF. Then, a new Bode diagram has to be drawn. With the newly detected cutoff frequency \( f_{c1} \) the value of the resistor \( R_x \) can be calculated. For this, Eq.(1.19) is used.

8) For an inverting amplifier according to Fig. 11, calculate the compensating capacitor \( C_k \) and the resistors \( R_0/R_1 \) for a DC gain of 18.3 dB. The phase reserve should be 65°.